Technical Comments

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Comment on "Analytical Prediction of Vortex Lift"

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SEVERAL comments are warranted concerning the paper by J.W. Purvis entitled "Analytical Prediction of Vortex Lift," which appears in the April 1981 issue of the *Journal of Aircraft*. The first comment is relative to the use of singular loadings in the design of mean camber shapes for wings experiencing leading-edge-vortex separated flow. By comparison, Ref. 1 outlines a low-drag, separated-flow design procedure which uses a more appropriate attached-flow initial loading that is nonsingular. There the final solution for the mean camber surface is obtained by *iterating* to find the shape-yielding minimum separated-flow drag at the required lift coefficient.

The second comment is that the section-induced drag, see Eq. (12), does not have an elliptical distribution *over the wing* as proposed, but only in the Trefftz plane. This is well known (e.g., Ref. 2), and means that for most wings of interest the section-induced drag has to be determined in the near field. The consequence of this wrong assumption leads to an error in Eq. (26) and, hence, in the magnitude of the leading-edge suction force and vortex flow aerodynamics for wings with varying leading-edge sweep.

The third comment deals with the missing multiplication by 2 in Eqs. (27), (30), (33), and (34), the missing division by 2 in Eqs. (28) and (37), and the typographical error in Eq. (32). Equation (32) should read

$$E_I = \frac{I}{\pi/2 - 2/3} \cong 1.106036$$

The missing multiplication by 2 in Eqs. (27), (30), (33), and (34) is just canceled by the missing division by 2 in the definition for $H(\eta)$. However, this is not the case in Eq. (37). From Ref. 3, Eq. (37) should be of the form

$$s(x) = \rho\pi(b/2)\lim_{\eta\to I}(I-\eta)v^2 \equiv \rho\pi(b/2)\lim_{\eta\to I}(I-\eta)(\delta^2/4)$$

This error in Eq. (37) will have the effect of making the side-edge suction force and the associated vortex-lift/moment too large by a factor of 2.

As a consequence of the foregoing comments, the resulting agreement or disagreement of this method with data for wings of varying leading-edge sweep and for wings with side edges is unconvincing.

References

[1] Lamar, J.E., Schemensky, R.T., and Reddy, C.S., "Development of a Vortex-Lift Design Procedure and Application to a Slender Maneuver Wing Configuration," *Journal of Aircraft*, Vol. 18, April 1981, pp. 259-266.

[2] Garner, H.C., "Some Remarks on Vortex Drag and Its Spanwise Distribution in Incompressible Flow," British NPL Aero Note 1048, 1966.

[3] von Kármán, T. and Burgers, J.M., "General Aerodynamic Theory-Perfect Fluids," *Aerodynamic Theory*, Vol. II, Division E, Sec. 10, Julius Springer, Berlin, 1935, pp. 48-53.

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Reply by the Author to J. E. Lamar

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THE author would like to thank Mr. Lamar for his comments concerning "Analytical Prediction of Vortex Lift."[1] There are several typographical errors in the paper which may have contributed to his confusion. With respect to Eq. (24), Eq. (25) obviously should read

$$cc_s\cos\Lambda = \frac{b}{2}E_0\int_0^\eta (cc_l\sin\alpha - cc_d\cos\alpha)\,d\eta_0$$

In Eq. (26), w_i should be w_0, as shown by direct substitution from Eqs. (2) and (12). Similarly, the required substitutions between Eqs. (26) and (27) show that Eq. (28) should obviously read

$$H(\eta) = 2\int_0^\eta \sqrt{I-\eta_0^2}\,d\eta_0 = \eta\sqrt{I-\eta^2} + \arcsin(\eta)$$

Finally, as noted by Mr. Lamar, Eq. (32) should read

$$E_I = \frac{I}{\pi/2 - 2/3} \cong 1.106036$$

The author does not understand the comparison mentioned in Mr. Lamar's first comment, since there is no design procedure presented in Ref. 1. The nonsingular attached-flow initial loading statement is also unclear, since the leading edges of thin wings, whether cambered or not, always exhibit theoretical pressure loading singularities in linearized potential flow. Vortex lattice schemes, of course, will only approach this behavior in the limit as the number of chordwise vortices becomes large.

Even after the extensive review and rebuttal process, Mr. Lamar still seems to be confused by the section of the paper dealing with the spanwise distribution of vortex lift. Garner[2] computes section drag from the small angle formula

$$cc_{d_i} = cc_l\alpha - \tfrac{1}{8}\pi\sec\Lambda\lim_{x\to x_{\rm LE}}(\Delta C_p)^2(x - x_{\rm LE})$$

where the second term on the right-hand side is the local leading-edge suction (acting in the plane of the wing). But the fundamental assumption of the suction analogy is that the leading-edge suction force acts *normal* to the chord, whereupon the section drag tends to that given by the usual Trefftz plane calculation. Under the assumption of an elliptic lift distribution, the Trefftz plane section drag formula is then Eq. (12). The accuracy of Eq. (26) [and (27)] is shown in Fig. 1, where the distribution given by Eq. (27) is compared with numerical results by Snyder and Lamar[3] for a low aspect ratio delta wing. The distributions are remarkably close, considering the number of "errors" and wrong "assumptions" in the present analysis.

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